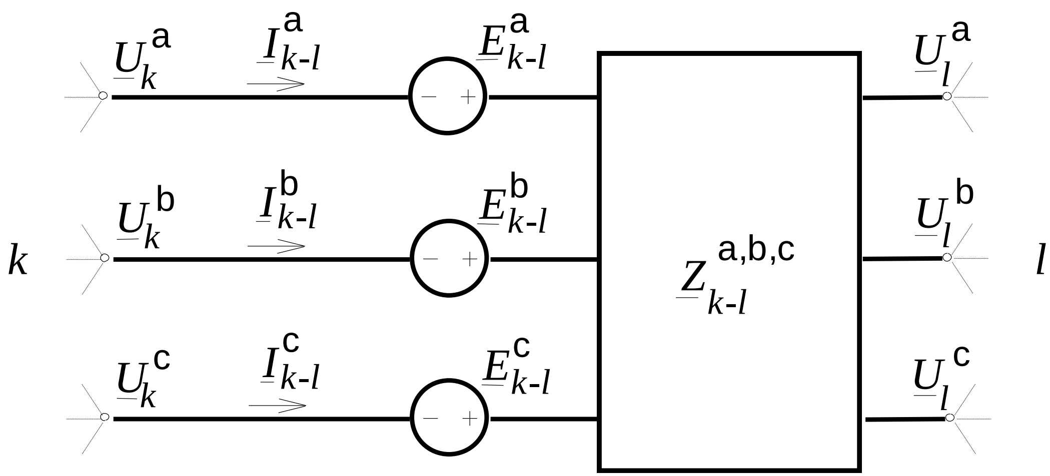
# МАТРИЦИ НА ТРАНСФОРМАЦИЈА




$$\underline{U}_{k\text{-}l}^{a} = \underline{U}_k^{a} - \underline{U}_l^{a};\quad \underline{U}_{k\text{-}l}^{b} = \underline{U}_k^{b} - \underline{U}_l^{b};\quad \underline{U}_{k\text{-}l}^{c} = \underline{U}_k^{c} - \underline{U}_l^{c}$$

$$\underline{Z}_{k-l}^{a,b,c} = \begin{bmatrix} \underline{Z}_{k-l}^{aa} & \underline{Z}_{k-l}^{ab} & \underline{Z}_{k-l}^{ac} \\ \underline{Z}_{k-l}^{ba} & \underline{Z}_{k-l}^{bb} & \underline{Z}_{k-l}^{bc} \\ \underline{Z}_{k-l}^{ca} & \underline{Z}_{k-l}^{cb} & \underline{Z}_{k-l}^{cc} \end{bmatrix}$$

$$\underline{U}_{k-l}^{a,b,c} + \underline{E}_{k-l}^{a,b,c} = \underline{Z}_{k-l}^{a,b,c} \cdot \underline{I}_{k-l}^{a,b,c}$$

- $\boldsymbol{T}_{3\times3}$ – матрица на трансформација на големините од трифазниот систем a,b,c во системот o,p,q

$$\underline{U}_{k-l}^{\text{a,b,c}} = \underline{\boldsymbol{T}} \cdot \underline{U}_{k-l}^{\text{o,p,q}} \qquad \underline{U}_{k}^{\text{a,b,c}} = \underline{\boldsymbol{T}} \cdot \underline{U}_{k}^{\text{o,p,q}} \qquad \underline{E}_{k-l}^{\text{a,b,c}} = \underline{\boldsymbol{T}} \cdot \underline{E}_{k-l}^{\text{o,p,q}}$$

$$\underline{I}_{k-l}^{\text{a,b,c}} = \underline{\boldsymbol{T}} \cdot \underline{I}_{k-l}^{\text{o,p,q}} \qquad \underline{I}_{k}^{\text{a,b,c}} = \underline{\boldsymbol{T}} \cdot \underline{I}_{k}^{\text{o,p,q}}$$

$$\underline{S}_{k-l_{(\text{a,b,c})}} = \underline{S}_{k-l_{(\text{o,p,q})}}$$

$$\underline{S}_{k-l_{(\text{a,b,c})}} = \left(\underline{U}_{k-l}^{\text{a,b,c}}\right)^{\mathrm{T}} \cdot \left(\underline{I}_{k-l}^{\text{a,b,c}}\right)^{*} \qquad \underline{S}_{k-l_{(\text{o,p,q})}} = \left(\underline{U}_{k-l}^{\text{o,p,q}}\right)^{\mathrm{T}} \cdot \left(\underline{I}_{k-l}^{\text{o,p,q}}\right)^{*}$$

$$\underline{S}_{k-l_{(\text{a,b,c})}} = \left(\underline{U}_{k-l}^{\text{o,p,q}}\right)^{\mathrm{T}} \cdot \underline{\boldsymbol{T}}^{\mathrm{T}} \cdot \underline{\boldsymbol{T}}^{*} \cdot \left(\underline{I}_{k-l}^{\text{o,p,q}}\right)^{*}$$

$$\underline{\boldsymbol{T}}^{\mathrm{T}} \cdot \underline{\boldsymbol{T}}^{*} = \boldsymbol{E}$$

$$\underline{\boldsymbol{T}}^{\mathrm{T}} \cdot \underline{\boldsymbol{T}}^{*} = \left(\underline{\boldsymbol{T}}^{*}\right)^{\mathrm{T}} \cdot \underline{\boldsymbol{T}} = \left(\underline{\boldsymbol{T}}^{\mathrm{T}}\right)^{*} \cdot \underline{\boldsymbol{T}} = \boldsymbol{E}$$

$$\underline{\boldsymbol{T}}^{-1} = \left(\underline{\boldsymbol{T}}^{*}\right)^{\mathrm{T}}$$ – унитарна матрица

$$\underline{U}_{k-l}^{\text{a,b,c}} + \underline{E}_{k-l}^{\text{a,b,c}} = \underline{Z}_{k-l}^{\text{a,b,c}} \cdot \underline{I}_{k-l}^{\text{a,b,c}}$$

$$\underline{\boldsymbol{T}} \cdot \left(\underline{U}_{k-l}^{\text{o,p,q}} + \underline{E}_{k-l}^{\text{o,p,q}}\right) = \underline{Z}_{k-l}^{\text{a,b,c}} \cdot \underline{\boldsymbol{T}} \cdot \underline{I}_{k-l}^{\text{o,p,q}} \quad / \left(\boldsymbol{T}^{*}\right)^{\mathrm{T}} \cdot$$

$$\underline{\boldsymbol{T}}^{-1} = \left(\underline{\boldsymbol{T}}^{*}\right)^{\mathrm{T}}$$

$$\underline{U}_{k-l}^{\text{o,p,q}} + \underline{E}_{k-l}^{\text{o,p,q}} = \left(\underline{\boldsymbol{T}}^{*}\right)^{\mathrm{T}} \cdot \underline{Z}_{k-l}^{\text{a,b,c}} \cdot \underline{\boldsymbol{T}} \cdot \underline{I}_{k-l}^{\text{o,p,q}}$$

$$\underline{U}_{k-l}^{\text{o,p,q}} + \underline{E}_{k-l}^{\text{o,p,q}} = \underline{Z}_{k-l}^{\text{o,p,q}} \cdot \underline{I}_{k-l}^{\text{o,p,q}}$$

$$\underline{Z}_{k-l}^{\text{o,p,q}} = \left(\underline{\boldsymbol{T}}^{*}\right)^{\mathrm{T}} \cdot \underline{Z}_{k-l}^{\text{a,b,c}} \cdot \underline{\boldsymbol{T}} \qquad \underline{Z}_{ik}^{\text{o,p,q}} = \left(\underline{\boldsymbol{T}}^{*}\right)^{\mathrm{T}} \cdot \underline{Z}_{ik}^{\text{a,b,c}} \cdot \underline{\boldsymbol{T}}$$

$$\underline{Y}_{k-l}^{\text{o,p,q}} = \left(\underline{\boldsymbol{T}}^{*}\right)^{\mathrm{T}} \cdot \underline{Y}_{k-l}^{\text{a,b,c}} \cdot \underline{\boldsymbol{T}} \qquad \underline{Y}_{ik}^{\text{o,p,q}} = \left(\underline{\boldsymbol{T}}^{*}\right)^{\mathrm{T}} \cdot \underline{Y}_{ik}^{\text{a,b,c}} \cdot \underline{\boldsymbol{T}}$$

$$\underline{\boldsymbol{T}} = \underline{\boldsymbol{T}}_{\text{сим.}} = \underline{\boldsymbol{T}}_{012\to\text{abc}} = \frac{1}{\sqrt{3}}\begin{bmatrix} 1 & 1 & 1 \\ 1 & \underline{a}^{2} & \underline{a} \\ 1 & \underline{a} & \underline{a}^{2} \end{bmatrix} \quad \underline{a} = \mathrm{e}^{\mathrm{j}\frac{2\pi}{3}} \quad \underline{a}^{2} = \mathrm{e}^{\mathrm{j}\frac{4\pi}{3}} \quad \underline{a}^{3} = \mathrm{e}^{\mathrm{j}\frac{6\pi}{3}} = \mathrm{e}^{\mathrm{j}2\pi} = 1 \quad \underline{a}^{4} = \mathrm{e}^{\mathrm{j}\frac{8\pi}{3}} = \mathrm{e}^{\mathrm{j}\frac{2\pi}{3}} = \underline{a}$$

$$1 + \underline{a} + \underline{a}^{2} = 0 \qquad \underline{a}^{*} = \underline{a}^{2} \qquad \underline{\boldsymbol{T}}_{012\to\text{abc}}^{-1} = \underline{\boldsymbol{T}}_{012\to\text{abc}}^{*} = \underline{\boldsymbol{T}}_{\text{abc}\to 012}$$

$$\begin{bmatrix} \underline{A}^{\text{a}} \\ \underline{A}^{\text{b}} \\ \underline{A}^{\text{c}} \end{bmatrix} = \underline{\boldsymbol{T}}_{012\to\text{abc}} \cdot \begin{bmatrix} \underline{A}^{0} \\ \underline{A}^{1} \\ \underline{A}^{2} \end{bmatrix} \qquad \underline{\boldsymbol{T}}_{\text{сим.}}^{-1} = \underline{\boldsymbol{T}}_{\text{сим.}}^{*} = \underline{\boldsymbol{T}}^{*} = \underline{\boldsymbol{T}}_{\text{abc}\to 012} = \frac{1}{\sqrt{3}}\begin{bmatrix} 1 & 1 & 1 \\ 1 & \underline{a} & \underline{a}^{2} \\ 1 & \underline{a}^{2} & \underline{a} \end{bmatrix} \qquad \begin{bmatrix} \underline{A}^{0} \\ \underline{A}^{1} \\ \underline{A}^{2} \end{bmatrix} = \underline{\boldsymbol{T}}_{\text{abc}\to 012} \cdot \begin{bmatrix} \underline{A}^{\text{a}} \\ \underline{A}^{\text{b}} \\ \underline{A}^{\text{c}} \end{bmatrix}$$

$$\underline{Z}_{k-l}^{\text{o,p,q}} = \left(\underline{\boldsymbol{T}}^{*}\right)^{\mathrm{T}} \cdot \underline{Z}_{k-l}^{\text{a,b,c}} \cdot \underline{\boldsymbol{T}}$$

ако гранката е вод или трансформатор

$$\underline{Z}_{k-l}^{0,1,2} = \frac{1}{\sqrt{3}}\begin{bmatrix} 1 & 1 & 1 \\ 1 & \underline{a} & \underline{a}^2 \\ 1 & \underline{a}^2 & \underline{a} \end{bmatrix} \cdot \begin{bmatrix} \underline{Z}_{k-l}^{\text{s}} & \underline{Z}_{k-l}^{\text{m}} & \underline{Z}_{k-l}^{\text{m}} \\ \underline{Z}_{k-l}^{\text{m}} & \underline{Z}_{k-l}^{\text{s}} & \underline{Z}_{k-l}^{\text{m}} \\ \underline{Z}_{k-l}^{\text{m}} & \underline{Z}_{k-l}^{\text{m}} & \underline{Z}_{k-l}^{\text{s}} \end{bmatrix} \cdot \frac{1}{\sqrt{3}}\begin{bmatrix} 1 & 1 & 1 \\ 1 & \underline{a}^2 & \underline{a} \\ 1 & \underline{a} & \underline{a}^2 \end{bmatrix}$$

$$\underline{Z}_{k-l}^{0,1,2} = \begin{bmatrix} \underline{Z}_{k-l}^{\text{s}} + 2\underline{Z}_{k-l}^{\text{m}} & 0 & 0 \\ 0 & \underline{Z}_{k-l}^{\text{s}} - \underline{Z}_{k-l}^{\text{m}} & 0 \\ 0 & 0 & \underline{Z}_{k-l}^{\text{s}} - \underline{Z}_{k-l}^{\text{m}} \end{bmatrix}$$

$\underline{Z}_{k-l}^{\text{s}} + 2\underline{Z}_{k-l}^{\text{m}} = \underline{Z}_{k-l}^{(0)}$ – импеданција на гранката *k*–*l* за нулти редослед

$\underline{Z}_{k-l}^{\text{s}} - \underline{Z}_{k-l}^{\text{m}} = \underline{Z}_{k-l}^{(1)}$ – импеданција на гранката *k*–*l* за директен редослед

$\underline{Z}_{k-l}^{\text{s}} - \underline{Z}_{k-l}^{\text{m}} = \underline{Z}_{k-l}^{(2)}$ – импеданција на гранката *k*–*l* за инверзен редослед

Ако гранката е електрична машина

$$\underline{Z}_{k-l}^{\text{a,b,c}} = \begin{bmatrix} \underline{Z}_{k-l}^{\text{s}} & \underline{Z}_{k-l}^{\text{m1}} & \underline{Z}_{k-l}^{\text{m2}} \\ \underline{Z}_{k-l}^{\text{m2}} & \underline{Z}_{k-l}^{\text{s}} & \underline{Z}_{k-l}^{\text{m1}} \\ \underline{Z}_{k-l}^{\text{m1}} & \underline{Z}_{k-l}^{\text{m2}} & \underline{Z}_{k-l}^{\text{s}} \end{bmatrix} \qquad \underline{\boldsymbol{T}} = \underline{\boldsymbol{T}}_{\text{сим.}} = \frac{1}{\sqrt{3}} \begin{bmatrix} 1 & 1 & 1 \\ 1 & \underline{a}^2 & \underline{a} \\ 1 & \underline{a} & \underline{a}^2 \end{bmatrix}$$

$$\underline{Z}_{k-l}^{0,1,2} = \left(\underline{\boldsymbol{T}}^*\right)^{\text{T}} \cdot \underline{Z}_{k-l}^{\text{a,b,c}} \cdot \underline{\boldsymbol{T}}$$

$$\underline{Z}_{k-l}^{0,1,2} = \begin{bmatrix} \underline{Z}_{k-l}^{\text{s}} + \underline{Z}_{k-l}^{\text{m1}} + \underline{Z}_{k-l}^{\text{m2}} & 0 & 0 \\ 0 & \underline{Z}_{k-l}^{\text{s}} + \underline{a}^2 \cdot \underline{Z}_{k-l}^{\text{m1}} + \underline{a} \cdot \underline{Z}_{k-l}^{\text{m2}} & 0 \\ 0 & 0 & \underline{Z}_{k-l}^{\text{s}} + \underline{a} \cdot \underline{Z}_{k-l}^{\text{m1}} + \underline{a}^2 \cdot \underline{Z}_{k-l}^{\text{m2}} \end{bmatrix}$$

$\underline{Z}_{k-l}^{\text{s}} + \underline{Z}_{k-l}^{\text{m1}} + \underline{Z}_{k-l}^{\text{m2}} = \underline{Z}_{k-l}^{(0)}$ – импеданција на гранката *k*–*l* за нулти редослед

$\underline{Z}_{k-l}^{\text{s}} + \underline{a}^2 \cdot \underline{Z}_{k-l}^{\text{m1}} + \underline{a} \cdot \underline{Z}_{k-l}^{\text{m2}} = \underline{Z}_{k-l}^{(1)}$ – импеданција на гранката *k*–*l* за директен редослед

$\underline{Z}_{k-l}^{\text{s}} + \underline{a} \cdot \underline{Z}_{k-l}^{\text{m1}} + \underline{a}^2 \cdot \underline{Z}_{k-l}^{\text{m2}} = \underline{Z}_{k-l}^{(2)}$ – импеданција на гранката *k*–*l* за инверзен редослед

$$\underline{U}_k^{\text{a,b,c}} = \underline{\boldsymbol{T}} \cdot \underline{U}_k^{0,1,2} \qquad / \underline{\boldsymbol{T}}^* \cdot \qquad\qquad \underline{\boldsymbol{T}}^* \cdot \underline{U}_k^{\text{a,b,c}} = \underline{U}_k^{0,1,2}$$

$$\underline{I}_k^{\text{a,b,c}} = \underline{\boldsymbol{T}} \cdot \underline{I}_k^{0,1,2} \qquad\qquad \underline{\boldsymbol{T}}^* \cdot \underline{I}_k^{\text{a,b,c}} = \underline{I}_k^{0,1,2}$$

$\underline{\boldsymbol{T}}$ – трансформација на фазните големини во симетрични големини

$\underline{\boldsymbol{T}}^{-1} = \underline{\boldsymbol{T}}^*$ – трансформација на големините од симетрични компоненти во фазни големини

За урамнотежен систем:

$$\underline{U}_i^{\text{a,b,c}} = \begin{bmatrix} \underline{U}_i^{\text{a}} \\ \underline{U}_i^{\text{b}} \\ \underline{U}_i^{\text{c}} \end{bmatrix} = \begin{bmatrix} 1 \\ \underline{a}^2 \\ \underline{a} \end{bmatrix} \cdot \underline{U}_i^{\text{a}}$$

$$\underline{U}_i^{0,1,2} = \underline{\boldsymbol{T}}^* \cdot \underline{U}_i^{\text{a,b,c}} = \frac{1}{\sqrt{3}} \begin{bmatrix} 1 & 1 & 1 \\ 1 & \underline{a} & \underline{a}^2 \\ 1 & \underline{a}^2 & \underline{a} \end{bmatrix} \cdot \begin{bmatrix} 1 \\ \underline{a}^2 \\ \underline{a} \end{bmatrix} \cdot \underline{U}_i^{\text{a}} = \begin{bmatrix} 0 \\ \sqrt{3}\underline{U}_i^{\text{a}} \\ 0 \end{bmatrix}$$