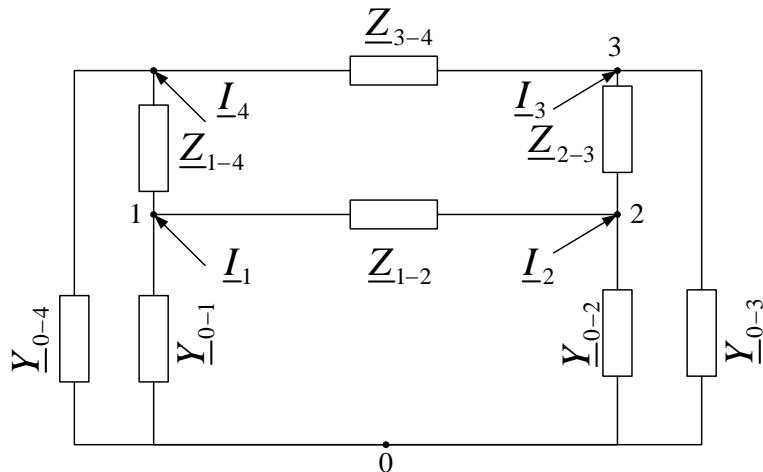


Матрица на поврзување (инциденција)

- Пример 2.8

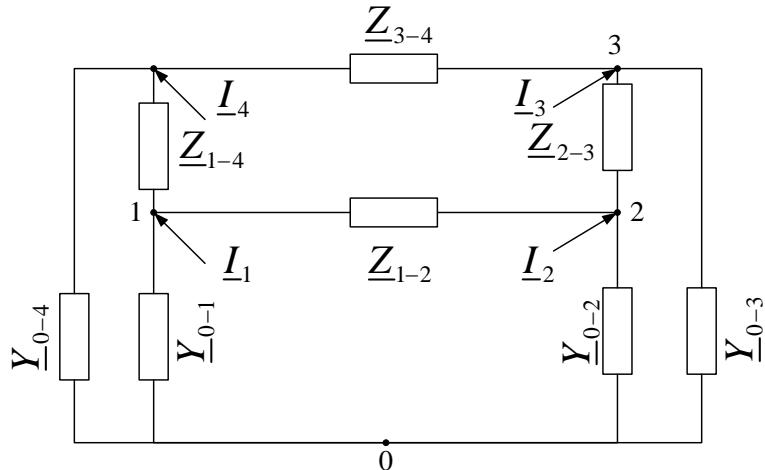


$$D = \begin{bmatrix} & 1 & 2 & 3 & 4 \\ 1-2 & 1 & -1 & 0 & 0 \\ 1-4 & 1 & 0 & 0 & -1 \\ 2-3 & 0 & 1 & -1 & 0 \\ 3-4 & 0 & 0 & 1 & -1 \\ 0-1 & -1 & 0 & 0 & 0 \\ 0-2 & 0 & -1 & 0 & 0 \\ 0-3 & 0 & 0 & -1 & 0 \\ 0-4 & 0 & 0 & 0 & -1 \end{bmatrix}$$

		$\Omega/\mu\text{S}$
1	\underline{Z}_{1-2}	$1.2+4i$
2	\underline{Z}_{1-4}	$2.4+8i$
3	\underline{Z}_{2-3}	$3.6+12i$
4	\underline{Z}_{3-4}	$2.4+8i$
5	\underline{Y}_{0-1}	$0+39i$
6	\underline{Y}_{0-2}	$0+52i$
7	\underline{Y}_{0-3}	$0+65i$
8	\underline{Y}_{0-4}	$0+52i$

Матрица на адмитанции на граници

- Пример 2.8



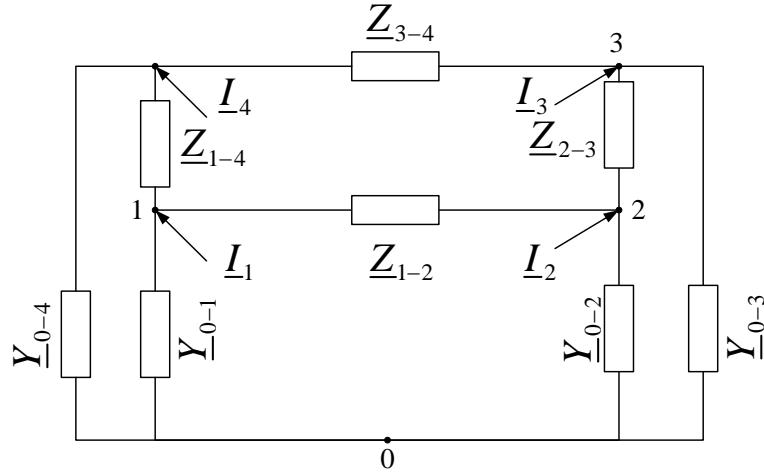
$$\underline{Y}_{i-j} = \frac{1}{Z_{i-j}}$$

		$\Omega/\mu\text{S}$
1	\underline{Z}_{1-2}	$1.2+4i$
2	\underline{Z}_{1-4}	$2.4+8i$
3	\underline{Z}_{2-3}	$3.6+12i$
4	\underline{Z}_{3-4}	$2.4+8i$
5	\underline{Y}_{0-1}	$0+39i$
6	\underline{Y}_{0-2}	$0+52i$
7	\underline{Y}_{0-3}	$0+65i$
8	\underline{Y}_{0-4}	$0+52i$

$$\underline{Y}_{\text{граници}} = \begin{bmatrix} \underline{Y}_{1-2} & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \underline{Y}_{1-4} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \underline{Y}_{2-3} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \underline{Y}_{3-4} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \underline{Y}_{0-1} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & \underline{Y}_{0-2} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & \underline{Y}_{0-3} \\ 0 & 0 & 0 & 0 & 0 & 0 & \underline{Y}_{0-4} \end{bmatrix}$$

Матрица на адмитанции

- Пример 2.8



$$\underline{Y} = \underline{D}^T \times \underline{Y}_{\text{гранки}} \times \underline{D}$$

$$\underline{Y}_{\text{гранки}} = \underline{Z}_{\text{гранки}}^{-1} = \begin{bmatrix} \underline{Z}_{1-2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \underline{Z}_{1-4} & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \underline{Z}_{2-3} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \underline{Z}_{3-4} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{\underline{Y}_{0-1}} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{1}{\underline{Y}_{0-2}} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{\underline{Y}_{0-3}} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{\underline{Y}_{0-4}} \end{bmatrix}^{-1}$$

		$\Omega/\mu\text{S}$
1	\underline{Z}_{1-2}	$1.2+4i$
2	\underline{Z}_{1-4}	$2.4+8i$
3	\underline{Z}_{2-3}	$3.6+12i$
4	\underline{Z}_{3-4}	$2.4+8i$
5	\underline{Y}_{0-1}	$0+39i$
6	\underline{Y}_{0-2}	$0+52i$
7	\underline{Y}_{0-3}	$0+65i$
8	\underline{Y}_{0-4}	$0+52i$

• Пример 2.8

Матрица на адмитанции

$$\underline{Y} = \underline{D}^T \times \underline{Y}_{\text{гранки}} \times \underline{D}$$

The screenshot shows the MATLAB interface with the following details:

- Workspace:** Shows variables **D**, **Y**, and **Ygr**.

Name	Value	Class
D	<8x4 double>	double
Y	<4x4 double>	double (comp)
Ygr	<8x8 double>	double (comp)
- Array Editor:** Displays two matrices:
 - D:** An 8x4 matrix with values:

	1	2	3	4	5	6	7	8	9	10
1	1	-1	0	0						
2	1	0	0	-1						
3	0	1	-1	0						
4	0	0	1	-1						
5	-1	0	0	0						
6	0	-1	0	0						
7	0	0	-1	0						
8	0	0	0	-1						
9										
10										
 - Ygr:** An 8x8 matrix with complex values:

	1	2	3	4	5	6	7	8	9	10
1	0.0688 - 0...	0 + 0i	0 + 0i	0 + 0i	0 + 0i	0 + 0i	0 + 0i	0 + 0i		
2	0 + 0i	0.0344 - 0...	0 + 0i	0 + 0i	0 + 0i	0 + 0i	0 + 0i	0 + 0i		
3	0 + 0i	0 + 0i	0.0229 - 0...	0 + 0i	0 + 0i	0 + 0i	0 + 0i	0 + 0i		
4	0 + 0i	0 + 0i	0 + 0i	0.0344 - 0...	0 + 0i	0 + 0i	0 + 0i	0 + 0i		
5	0 + 0i	0 + 0i	0 + 0i	0 + 0i	0 + 0i	0 + 0i	0 + 0i	0 + 0i		
6	0 + 0i	0 + 0i	0 + 0i	0 + 0i	0 + 0i	0 + 0.0001i	0 + 0i	0 + 0i		
7	0 + 0i	0 + 0i	0 + 0i	0 + 0i	0 + 0i	0 + 0i	0 + 0.0001i	0 + 0i		
8	0 + 0i	0 + 0i	0 + 0i	0 + 0i	0 + 0i	0 + 0i	0 + 0i	0 + 0.0001i		
9										
10										
11										
12										
- Command Window:** Shows the command `>> Y=D'*Ygr*D` and its output.

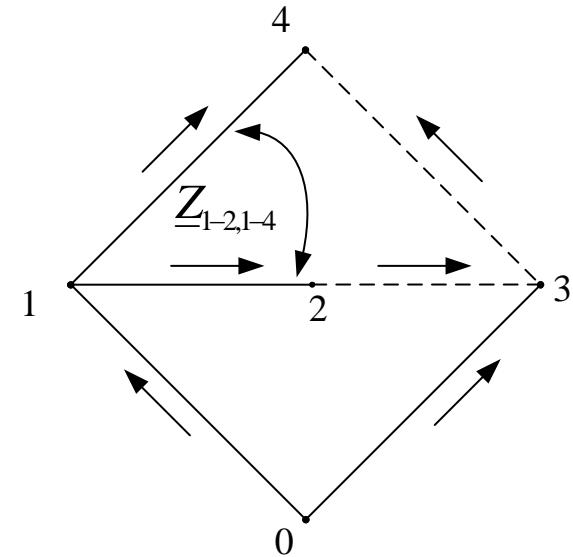

```
>> Y=D'*Ygr*D
Y =
    0.1032 - 0.3441i   -0.0688 + 0.2294i       0           -0.0344 + 0.1147i
   -0.0688 + 0.2294i    0.0917 - 0.3058i  -0.0229 + 0.0765i       0
           0   -0.0229 + 0.0765i  0.0573 - 0.1911i  -0.0344 + 0.1147i
   -0.0344 + 0.1147i        0   -0.0344 + 0.1147i  0.0688 - 0.2293i
>>
```

Матрица на адмитанции на ЕЕС

- Ако постои индуктивна спрега помеѓу гранките 1-2 и 1-4 матрицата $\underline{Y}_{\text{гранки}}$ не е дијагонална матрица

$$\underline{Y} = \mathbf{A}^T \cdot \begin{bmatrix} \underline{Z}_{0-1} & 0 & 0 & 0 & 0 & 0 \\ 0 & \underline{Z}_{1-2} & 0 & \underline{Z}_{1-2,1-4} & 0 & 0 \\ 0 & 0 & \underline{Z}_{0-3} & 0 & 0 & 0 \\ 0 & \underline{Z}_{1-2,1-4} & 0 & \underline{Z}_{1-4} & 0 & 0 \\ 0 & 0 & 0 & 0 & \underline{Z}_{2-3} & 0 \\ 0 & 0 & 0 & 0 & 0 & \underline{Z}_{3-4} \end{bmatrix}^{-1} \cdot \mathbf{A}$$

$$\underline{Y} = \begin{bmatrix} \underline{Y}_{11} & \underline{Y}_{12} & \underline{Y}_{13} & \underline{Y}_{14} \\ \underline{Y}_{21} & \underline{Y}_{22} & \underline{Y}_{23} & \underline{Y}_{24} \\ \underline{Y}_{31} & \underline{Y}_{32} & \underline{Y}_{33} & \underline{Y}_{34} \\ \underline{Y}_{41} & \underline{Y}_{42} & \underline{Y}_{43} & \underline{Y}_{44} \end{bmatrix}$$



Матрица на адмитанции на ЕЕС

- Пример

- реактанциите на гранките се дадени во р.и.

	1	2	3	4	
1	0.6	0.1	0	0.2	0
2	0.1	0.5	0	0	0
3	0	0	0.5	0	0
4	0.2	0	0	0.4	0
5	0	0	0	0	0.2

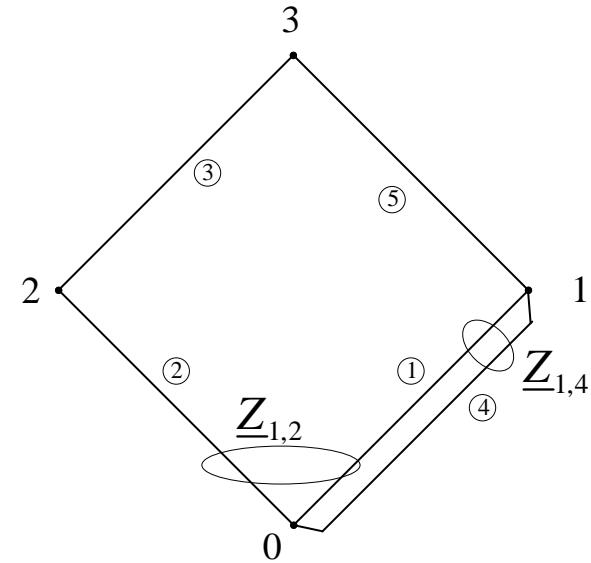
$\underline{Z}_{gr} = j$

	1	2	3
1	-1	0	0
2	0	-1	0
3	0	1	-1
4	-1	0	0
5	1	0	1

$A =$

	1	2	3	4	5
1	2.08333	-0.4167	0	-1.0417	0
2	-0.4167	2.08333	0	0.20833	0
3	0	0	2	0	0
4	-1.0417	0.20833	0	3.02083	0
5	0	0	0	0	5

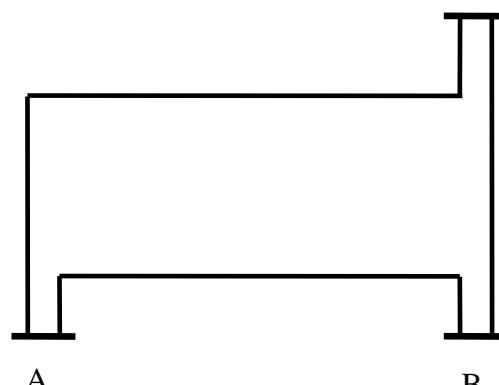
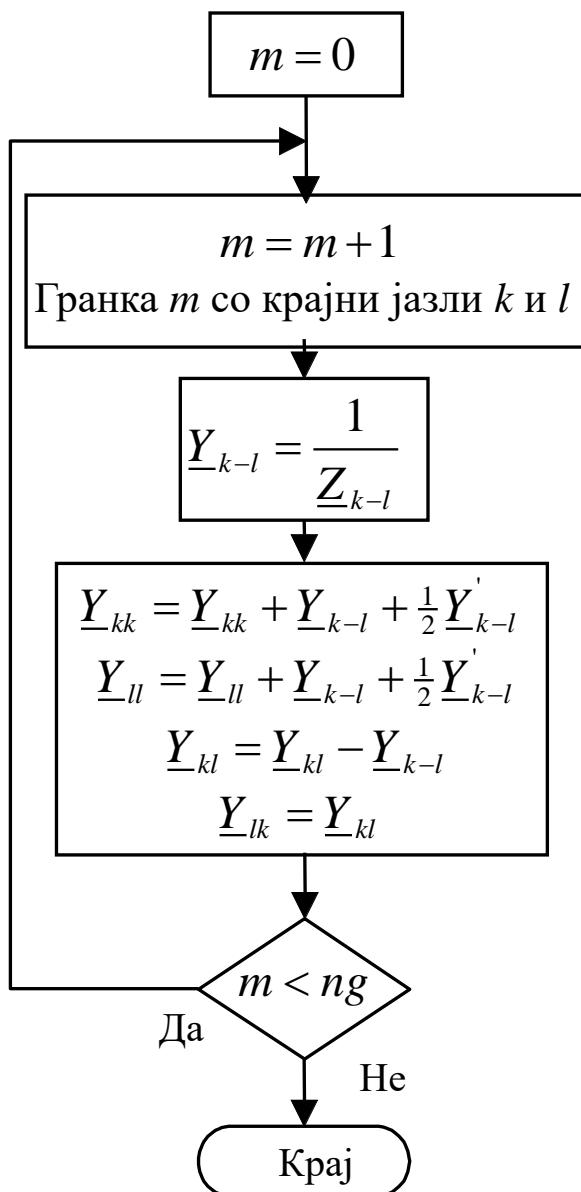
$\underline{Y}_{gr} = j$



$$\underline{Y} = \underline{A}^T \cdot \begin{bmatrix} \underline{Z}_1 & \underline{Z}_{1,2} & 0 & \underline{Z}_{1,4} & 0 \\ \underline{Z}_{1,2} & \underline{Z}_2 & 0 & 0 & 0 \\ 0 & 0 & \underline{Z}_3 & 0 & 0 \\ \underline{Z}_{1,4} & 0 & 0 & \underline{Z}_4 & 0 \\ 0 & 0 & 0 & 0 & \underline{Z}_5 \end{bmatrix}^{-1} \cdot \underline{A}$$

	1	2	3
1	8.02083	-0.2083	5
2	-0.2083	4.08333	-2
3	5	-2	7

Постапка за формирање на матрицата \underline{Y}



Податоци за гранките
(per unit)

Гранка	R	X	G'	B'
C-A	0,10	0,20	0	0,04
A-B	0,05	0,15	0	0,02
B-C	0,10	0,30	0	0,04

$$\underline{Y}_{kk} = \sum_{j \in \alpha_k} \underline{Y}_{k-j}$$

$$\underline{Y}_{kl} = \underline{Y}_{lk} = - \sum_{\beta_{k-l}} \underline{Y}_{k-l}$$

$$\underline{Y}_{C-A} = \frac{1}{\underline{Z}_{C-A}} = \frac{1}{0,1 + j0,2} = 2 - j4$$

$$\underline{Y}_{A-B} = \frac{1}{\underline{Z}_{A-B}} = \frac{1}{0,05 + j0,15} = 2 - j6$$

$$\underline{Y}_{B-C} = \frac{1}{\underline{Z}_{B-C}} = \frac{1}{0,1 + j0,3} = 1 - j3$$

$$\begin{aligned}\underline{Y}_{CC} &= \underline{Y}_{C-A} + \underline{Y}_{B-C} + \frac{1}{2} \cdot (\underline{Y}'_{C-A} + \underline{Y}'_{B-C}) \\ &= 2 - j4 + 1 - j3 + \frac{1}{2} \cdot (j0,04 + j0,04) = 3 - j6,96\end{aligned}$$

$$\begin{aligned}\underline{Y}_{AA} &= \underline{Y}_{C-A} + \underline{Y}_{A-B} + \frac{1}{2} \cdot (\underline{Y}'_{C-A} + \underline{Y}'_{A-B}) \\ &= 2 - j4 + 2 - j6 + \frac{1}{2} \cdot (j0,04 + j0,02) = 4 - j9,97\end{aligned}$$

$$\begin{aligned}\underline{Y}_{BB} &= \underline{Y}_{B-C} + \underline{Y}_{A-B} + \frac{1}{2} \cdot (\underline{Y}'_{B-C} + \underline{Y}'_{A-B}) \\ &= 1 - j3 + 2 - j6 + \frac{1}{2} \cdot (j0,04 + j0,02) = 3 - j8,97\end{aligned}$$

$$\underline{Y}_{CA} = \underline{Y}_{AC} = -\underline{Y}_{C-A} = -(2 - j4) = -2 + j4$$

$$\underline{Y}_{CB} = \underline{Y}_{BC} = -\underline{Y}_{B-C} = -(1 - j3) = -1 + j3$$

$$\underline{Y}_{AB} = \underline{Y}_{BA} = -\underline{Y}_{A-B} = -(2 - j6) = -2 + j6$$

Податоци за гранките
(per unit)

Гранка	R	X	G'	B'
C-A	0,10	0,20	0	0,04
A-B	0,05	0,15	0	0,02
B-C	0,10	0,30	0	0,04

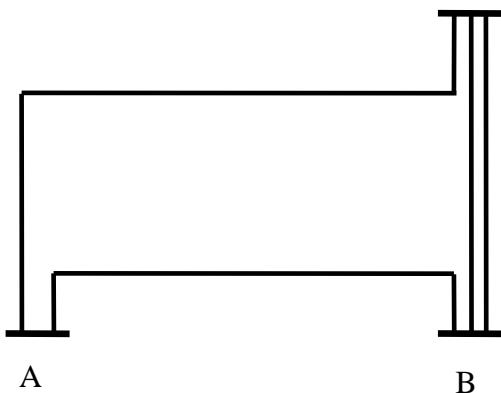
$$\underline{Y}_{C-A} = \frac{1}{Z_{C-A}} = \frac{1}{0,1 + j0,2} = 2 - j4$$

$$\underline{Y}_{A-B} = \frac{1}{Z_{A-B}} = \frac{1}{0,05 + j0,15} = 2 - j6$$

$$\underline{Y}_{B-C} = \frac{1}{Z_{B-C}} = \frac{1}{0,1 + j0,3} = 1 - j3$$

$$\underline{Y} = C \begin{bmatrix} B & C & A \\ 3 & -1 & -2 \\ -1 & 3 & -2 \\ -2 & -2 & 4 \end{bmatrix} + j \begin{bmatrix} B & C & A \\ -8,97 & 3,00 & 6,00 \\ 3,00 & -6,96 & 4,00 \\ 6,00 & 4,00 & -9,97 \end{bmatrix} \begin{bmatrix} B \\ C \\ A \end{bmatrix}$$

Постапка за формирање на матрицата \underline{Y}



$$\underline{Y}_{(B-C)_2} = \frac{1}{\underline{Z}_{(B-C)_2}} = \frac{1}{0,05 + j0,1} = 4 - j8$$

Податоци за гранките
(per unit)

Гранка	R	X	G'	B'
C-A	0,10	0,20	0	0,04
A-B	0,05	0,15	0	0,02
(B-C) ₁	0,10	0,30	0	0,04
(B-C) ₂	0,05	0,10	0	0,03

$$\underline{Y}_{AA} = \underline{Y}_{C-A} + \underline{Y}_{A-B} + \frac{1}{2} \cdot (\underline{Y}'_{C-A} + \underline{Y}'_{A-B}) = 4 - j9,97$$

$$\underline{Y}_{AB} = \underline{Y}_{BA} = -\underline{Y}_{A-B} = -(2 - j6) = -2 + j6 \quad \underline{Y}_{CA} = \underline{Y}_{AC} = -\underline{Y}_{C-A} = -(2 - j4) = -2 + j4$$

$$\underline{Y} = C \begin{bmatrix} B & C & A \\ -1 & 3 & -2 \\ A & -2 & -2 \end{bmatrix} + j \begin{bmatrix} -8,97 & 3,00 & 6,00 \\ 3,00 & -6,96 & 4,00 \\ 6,00 & 4,00 & -9,97 \end{bmatrix} \begin{bmatrix} B \\ C \\ A \end{bmatrix}$$

$$\begin{aligned}\underline{Y}_{BB} &= \underline{Y}_{(B-C)_1} + \underline{Y}_{(B-C)_2} + \underline{Y}_{A-B} + \frac{1}{2} \cdot (\underline{Y}'_{(B-C)_1} + \underline{Y}'_{(B-C)_2} + \underline{Y}'_{A-B}) \\ &= 1 - j3 + 4 - j8 + 2 - j6 + \frac{1}{2} \cdot (j0,04 + j0,03 + j0,02) = 7 - j16,955\end{aligned}$$

$$\begin{aligned}\underline{Y}_{CC} &= \underline{Y}_{C-A} + \underline{Y}_{(B-C)_1} + \underline{Y}_{(B-C)_2} + \frac{1}{2} \cdot (\underline{Y}'_{(B-C)_1} + \underline{Y}'_{(B-C)_2} + \underline{Y}'_{A-C}) \\ &= 2 - j4 + 1 - j3 + 4 - j8 + \frac{1}{2} \cdot (j0,04 + j0,03 + j0,04) = 7 - j14,945\end{aligned}$$

$$\underline{Y}_{CB} = \underline{Y}_{BC} = -\underline{Y}_{(B-C)_1} - \underline{Y}_{(B-C)_2} = -(1 - j3) - (4 - j8) = -5 + j11$$

$$\underline{Y} = C \begin{bmatrix} B & C & A \\ \hline B & \begin{bmatrix} 7 \\ -5 \end{bmatrix} & \begin{bmatrix} -2 \\ -2 \end{bmatrix} \\ \hline C & \begin{bmatrix} -5 \\ 7 \end{bmatrix} & \begin{bmatrix} -2 \\ 4 \end{bmatrix} \\ \hline A & \begin{bmatrix} -2 & -2 & 4 \end{bmatrix} \end{bmatrix} + j \begin{bmatrix} B & C & A \\ \hline -16,955 & \begin{bmatrix} 11,00 \\ 11,00 \end{bmatrix} & 6,00 \\ \hline 11,00 & \begin{bmatrix} -14,945 \\ 6,00 \end{bmatrix} & 4,00 \\ \hline -14,945 & 4,00 & -9,97 \end{bmatrix}$$

Нова гранка B-C се менуваат елементите \underline{Y}_{BB} , \underline{Y}_{CC} , \underline{Y}_{BC} и \underline{Y}_{CB}

Додавање нова гранка $k-l$ во системот

$$\underline{Y}_{kk} = \underline{Y}_{kk} + \underline{Y}_{k-l} + \frac{1}{2} \cdot \underline{Y}'_{k-l}$$

$$\underline{Y}_{ll} = \underline{Y}_{ll} + \underline{Y}_{k-l} + \frac{1}{2} \cdot \underline{Y}'_{k-l}$$

$$\underline{Y}_{kl} = \underline{Y}_{lk} = \underline{Y}_{kl} - \underline{Y}_{k-l}$$

Исклучување на гранката $k-l$

$$\underline{Y}_{kk} = \underline{Y}_{kk} - \underline{Y}_{k-l} - \frac{1}{2} \cdot \underline{Y}'_{k-l}$$

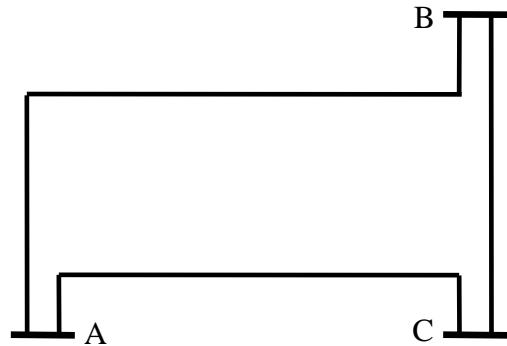
$$\underline{Y}_{ll} = \underline{Y}_{ll} - \underline{Y}_{k-l} - \frac{1}{2} \cdot \underline{Y}'_{k-l}$$

$$\underline{Y}_{kl} = \underline{Y}_{lk} = \underline{Y}_{kl} + \underline{Y}_{k-l}$$

Пресметка на матрицата на адмитанции

```
function [ng,nj,Y] = Ymat(tabla_granki)
%Ymat - funkcija za formiranje na Y
[ng,nn] = size(tabla_granki);
nj = max(max(tabla_granki(1:ng,1:2)));
Y=complex(zeros(nj),zeros(nj));
for k = 1:ng
    i = fix(tabla_granki(k,1));
    j = fix(tabla_granki(k,2));
    if i == 0 | j == 0
        g = tabla_granki(k,3);
        b = tabla_granki(k,4);
        ygr = complex(g,b) * 1.e-6;
    %podatocite za admitanciite vo tabelata za granki se dadeni vo mikroSimensi
        if i == 0
            Y(j,j) = Y(j,j) + ygr;
        else
            Y(i,i) = Y(i,i) + ygr;
        end %if
    else
        r = tabla_granki(k,3);
        x = tabla_granki(k,4);
        ygr = 1./complex(r,x);
        Y(i,i) = Y(i,i) + ygr;
        Y(j,j) = Y(j,j) + ygr;
        Y(i,j) = Y(i,j) - ygr;
        Y(j,i) = Y(i,j);
    end %if
end %for
```

Да се определи матрицата на адмитанции ако се исключи гранката А–В



$$\underline{Y} = \begin{bmatrix} 4,5 & -3,0 & -1,5 \\ -3,0 & 9,0 & -6,0 \\ -1,50 & -6,0 & 7,5 \end{bmatrix} + j \begin{bmatrix} -5,97 & 4,00 & 2,00 \\ 4,00 & -11,96 & 8,00 \\ 2,00 & 8,00 & -9,97 \end{bmatrix} \begin{matrix} B \\ C \\ A \end{matrix}$$

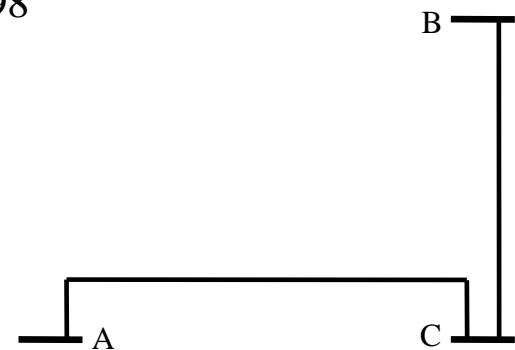
$$\underline{Y}_{A-B} = 1,5 - j2,0$$

$$\underline{Y}'_{A-B} = j0,02$$

$$\underline{Y}_{AA} = \underline{Y}_{AA} - \underline{Y}_{A-B} - \frac{1}{2} \cdot \underline{Y}'_{A-B} = 7,5 - j9,97 - 1,5 + j2,0 - j0,01 = 6,0 - j7,98$$

$$\underline{Y}_{BB} = \underline{Y}_{BB} - \underline{Y}_{A-B} - \frac{1}{2} \cdot \underline{Y}'_{A-B} = 4,5 - j5,97 - 1,5 + j2,0 - j0,01 = 3,0 - j3,98$$

$$\underline{Y}_{AB} = \underline{Y}_{BA} = \underline{Y}_{AB} + \underline{Y}_{A-B} = -1,5 + j2 + 1,5 - j2 = 0$$



$$\underline{Y} = \begin{bmatrix} 3,0 & -3,0 & 0 \\ -3,0 & 9,0 & -6,0 \\ 0 & -6,0 & 6,0 \end{bmatrix} + j \begin{bmatrix} -3,98 & 4,00 & 0 \\ 4,00 & -11,96 & 8,00 \\ 0 & 8,00 & -7,98 \end{bmatrix} \begin{matrix} B \\ C \\ A \end{matrix}$$

Од матрицата на адмитанции да се определат импеданциите на редните гранки

$$\underline{Y} = \begin{bmatrix} 3,0 & -3,0 & 0 \\ -3,0 & 9,0 & -6,0 \\ 0 & -6,0 & 6,0 \end{bmatrix} + j \begin{bmatrix} -3,98 & 4,00 & 0 \\ 4,00 & -11,96 & 8,00 \\ 0 & 8,00 & -7,98 \end{bmatrix} \begin{array}{l} \text{B} \\ \text{C} \\ \text{A} \end{array}$$

$$\underline{Y}_{CB} = \underline{Y}_{BC} = -\underline{Y}_{B-C_{ekv.}} = \frac{1}{\underline{Z}_{B-C_{ekv.}}} = -3,0 + j4,0 \Rightarrow \underline{Z}_{B-C_{ekv.}} = \frac{1}{-3,0 + j4,0} = 0,12 + j0,16$$

$$\underline{Y}_{AC} = \underline{Y}_{CA} = -\underline{Y}_{A-C_{ekv.}} = \frac{1}{\underline{Z}_{A-C_{ekv.}}} = -6,0 + j8,0 \Rightarrow \underline{Z}_{A-C_{ekv.}} = \frac{1}{-6,0 + j8,0} = 0,06 + j0,08$$

$$\underline{Y}_{AB} = -\underline{Y}_{A-B_{ekv.}} = \frac{1}{\underline{Z}_{A-B_{ekv.}}} = 0 + j0 \Rightarrow \underline{Z}_{A-B_{ekv.}} = \infty$$