4. Решавање на преодни процеси со примена на PSCAD

Текстот кој што следува е преземан од книгата Neville Watson and Jos Arrillaga, *Power Systems Electromagnetic Transients Simulation*, Published by: The Institution of Electrical Engineers, London, United Kingdom, 2003.

4.1. Introduction

A continuous function can be simulated by substituting a numerical integration formula into the differential equation and rearranging the function into an appropriate form. Among the factors to be taken into account in the selection of the numerical integrator are the error due to truncated terms, its properties as a differentiator, error propagation and frequency response.

Numerical integration substitution (NIS) constitutes the basis of Dommel's EMTP [l]–[3], which, as explained in the introductory chapter, is now the most generally accepted method for the solution of electromagnetic transients. The EMTP method is an integrated approach to the problems of:

- forming the network differential equations
- collecting the equations into a coherent system to be solved
- numerical solution of the equations.

The trapezoidal integrator (described in Appendix C) is used for the numerical integrator substitution, due to its simplicity, stability and reasonable accuracy in most circumstances. However, being based on a truncated Taylor's series, the trapezoidal rule can cause numerical oscillations under certain conditions due to the neglected terms [4]. This problem will be discussed further in Chapters 5 and 9.

The other basic characteristic of Dommel's method is the discretisation of the system components, given a predetermined time step, which are then combined in a solution for the nodal voltages. Branch elements are represented by the relationship which they maintain between branch current and nodal voltage.

This chapter describes the basic formulation and solution of the numerical integrator substitution method as implemented in the electromagnetic transient programs.

4.2. Discretisation of R, L, C Elements

4.2.1. Resistance

The simplest circuit element is a resistor connected between nodes *k* and *m*, as shown in Figure 4.1, and is represented by the equation:

$$i_{km}(t) = \frac{1}{R}(v_k(t) - v_m(t)) \tag{4.1}$$

Resistors are accurately represented in the EMTP formulation provided *R* is not too small. If the value of *R* is too small its inverse in the system matrix will be large, resulting in poor conditioning of the solution at every step. This gives inaccurate results due to the finite precision of numerical calculations. On the other hand, very large values of *R* do not degrade the overall solution. In EMTDC version 3 if *R* is below a threshold (the default threshold value is 0.0005) then *R* is automatically set to zero and a modified solution method used.

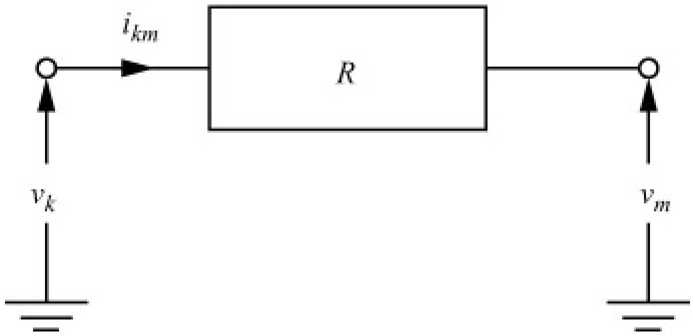


Figure 4.1: Resistor

4.2.2. Inductance

The differential equation for the inductor shown in Figure 4.2 is:

$$v_L = v_k - v_m = L\frac{di_{km}}{dt} \tag{4.2}$$

Rearranging:

$$i_{km(t)} = i_{km(t-\Delta t)} + \int_{t-\Delta t}^{t} (v_k - v_m)\, dt \tag{4.3}$$

Applying the trapezoidal rule gives:

$$i_{km(t)} = i_{km(t-\Delta t)} + \frac{\Delta t}{2L}((v_k - v_m)_{(t)} + (v_k - v_m)_{(t-\Delta t)}) \tag{4.4}$$

$$= i_{km(t-\Delta t)} + \frac{\Delta t}{2L}(v_{k(t-\Delta t)} - v_{m(t-\Delta t)}) + \frac{\Delta t}{2L}(v_{k(t)} - v_{m(t)}) \tag{4.5}$$

$$i_{km}(t) = I_{\text{History}}(t - \Delta t) + \frac{1}{R_{\text{eff}}}(v_k(t) - v_m(t)) \tag{4.6a}$$

This equation can be expressed in the form of a Norton equivalent (or companion circuit) as illustrated in Figure 4.3. The term relating the current contribution at the present time step to voltage at the present time step ($1/R_{\text{eff}}$) is a conductance (instantaneous term) and the contribution to current from the previous time step quantities is a current source (History term).

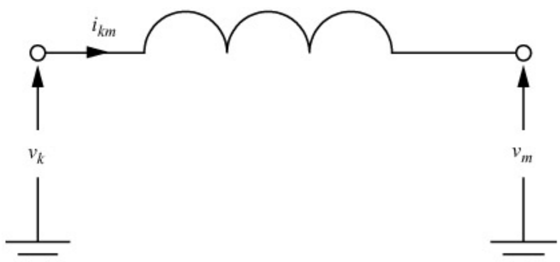


Figure 4.2: Inductor

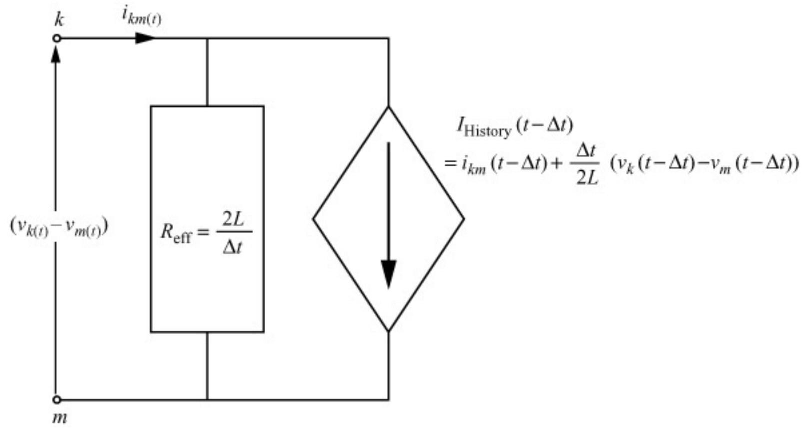


Figure 4.3: Norton equivalent of the inductor

In equation 4.6 $I_{History}(t - \Delta t) = i_{km}(t - \Delta t) + (\Delta t/2L)(v_k(t - \Delta t) - v_m(t - \Delta t))$ and

$$R_{eff} = \frac{2L}{\Delta t}. \tag{4.6b}$$

The term $2L/\Delta t$ is known as the instantaneous term as it relates the current to the voltage at the same time point, i.e. any change in one will instantly be reflected in the other. As an effective resistance, very small values of L or rather $2L/\Delta t$, can also result in poor conditioning of the conductance matrix.

Transforming equation 4.6 to the z-domain gives:

$$I_{km}(z) = z^{-1} I_{km}(z) + \frac{\Delta t}{2L}(1 + z^{-1})(V_k(z) - V_m(z))$$

Rearranging gives the following transfer between current and voltage in the z-domain:

$$\frac{I_{km}(z)}{(V_k(z) - V_m(z))} = \frac{\Delta t}{2L}\frac{(1 + z^{-1})}{(1 - z^{-1})} \tag{4.7}$$

4.2.3. Capacitance

With reference to Figure 4.4 the differential equation for the capacitor is:

$$i_{km}(t) = C\frac{d(v_k(t) - v_m(t))}{dt} \tag{4.8}$$

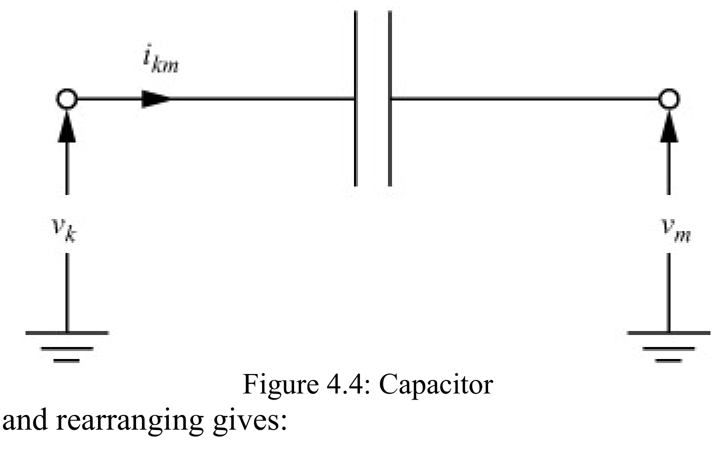


Figure 4.4: Capacitor

Integrating and rearranging gives:

$$v_{km(t)} = (v_{k(t)} - v_{m(t)}) = (v_{k(t-\Delta t)} - v_{m(t-\Delta t)}) + \frac{1}{C}\int_{t-\Delta t}^{t} i_{km}\, dt \tag{4.9}$$

and applying the trapezoidal rule:

$$v_{km}(t) = (v_k(t) - v_m(t)) = (v_k(t-\Delta t) - v_m(t-\Delta t)) + \frac{\Delta t}{2C}(i_{km}(t) + i_{km}(t-\Delta t)) \tag{4.10}$$

Hence the current in the capacitor is given by:

$$\begin{aligned} i_{km}(t) &= \frac{2C}{\Delta t}(v_k(t) - v_m(t)) - i_{km}(t-\Delta t) - \frac{2C}{\Delta t}(v_k(t-\Delta t) - v_m(t-\Delta t)) \\ &= \frac{1}{R_{\mathrm{eff}}}[v_k(t) - v_m(t)] + I_{\mathrm{History}}(t-\Delta t) \end{aligned} \tag{4.11}$$

which is again a Norton equivalent as depicted in Figure 4.5. The instantaneous term in equation 4.11 is:

$$R_{\mathrm{eff}} = \frac{\Delta t}{2C} \tag{4.12}$$

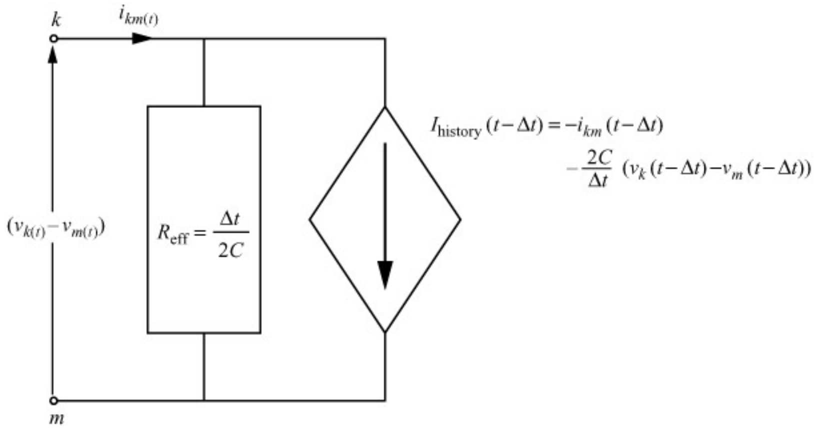


Figure 4.5: Norton equivalent of the capacitor

Thus very large values of *C*, although they are unlikely to be used, can cause ill conditioning of the conductance matrix.

The History term represented by a current source is:

$$I_{\mathrm{History}(t-\Delta t)} = -i_{km(t-\Delta t)} - \frac{2C}{\Delta t}(v_{k(t-\Delta t)} - v_{m(t-\Delta t)}) \tag{4.13}$$

Transforming to the *z*-domain gives:

$$I_{km} = -z^{-1} I_{km} - \frac{2C}{\Delta t}(V_k - V_m) z^{-1} + \frac{2C}{\Delta t}(V_k - V_m) \tag{4.14}$$

$$\frac{I_{km}}{(V_k - V_m)} = \frac{2C}{\Delta t}\frac{(1 - z^{-1})}{(1 + z^{-1})} \tag{4.15}$$

It should be noted that any implicit integration formula can be substituted into a differential equation to form a difference equation (and a corresponding Norton equivalent). Table 4.1 shows the Norton components that result from using three different integration methods.

Table 4.1: Norton components for different integration formulae

Integration method	R_{eq}	$I_{History}$
Inductor		
Backward Euler	$\frac{L}{\Delta t}$	i_{n-1}
Trapezoidal	$\frac{2L}{\Delta t}$	$i_{n-1}+\frac{\Delta t}{2L}v_{n-1}$
Gear 2nd order	$\frac{3L}{2\Delta t}$	$\frac{4}{3}i_{n-1}-\frac{1}{3}i_{n-2}$
Capacitor		
Backward Euler	$\frac{\Delta t}{C}$	$-\frac{C}{\Delta t}v_{n-1}$
Trapezoidal	$\frac{\Delta t}{2C}$	$-\frac{C}{\Delta t}v_{n-1}-i_{n-1}$
Gear 2nd order	$\frac{2\Delta t}{3C}$	$-\frac{2C}{\Delta t}v_{n-1}-\frac{C}{2\Delta t}v_{n-2}$

4.3. Dual Norton Model of the Transmission Line

A detailed description of transmission line modelling is deferred to Chapter 6. The single-phase lossless line [4] is used as an introduction at this stage, to illustrate the simplicity of Dommel's method.

Consider the lossless distributed parameter line depicted in Figure 4.8, where L' is the inductance and C' the capacitance per unit length. The wave propagation equations for this line are:

$$-\frac{\partial v(x,t)}{\partial x}=L'\frac{\partial i(x,t)}{\partial t} \tag{4.22}$$

$$-\frac{\partial i(x,t)}{\partial x}=C'\frac{\partial v(x,t)}{\partial t} \tag{4.23}$$

and the general solution:

$$i(x,t)=f_1(x-\varpi t)+f_2(x+\varpi t) \tag{4.24}$$

$$v(x,t)=Z\cdot f_1(x-\varpi t)-Z\cdot f_2(x+\varpi t) \tag{4.25}$$

with $f_1(x - \omega t)$ and $f_2(x + \omega t)$ being arbitrary functions of $(x - \omega t)$ and $(x + \omega t)$ respectively. $f_1(x - \omega t)$ represents a wave travelling at velocity ω in a forward direction (depicted in Figure 4.8) and $f_2(x + \omega t)$ a wave travelling in a backward direction. Z_C, the surge or characteristic impedance and ω, the phase velocity, are given by:

$$Z_C=\sqrt{\frac{L'}{C'}} \tag{4.26}$$

$$\varpi = \frac{1}{\sqrt{L'C'}} \tag{4.27}$$

Multiplying equation 4.24 by Z_C and adding it to, and subtracting it from, equation 4.25 leads to:

$$v(x,t) + Z_C \cdot i(x,t) = 2Z_C \cdot f_1(x - \varpi t) \tag{4.28}$$

$$v(x,t) - Z_C \cdot i(x,t) = -2Z_C \cdot f_2(x + \varpi t) \tag{4.29}$$

It should be noted that $v(x, t) + Z_C \cdot i(x, t)$ is constant when $(x - \omega t)$ is constant. If d is the length of the line, the travelling time from one end (k) to the other end (m) of the line to observe a constant $v(x, t) + Z_C \cdot i(x, t)$ is:

$$\tau = d/\varpi = d\sqrt{L'C'} \tag{4.30}$$

Hence

$$v_k(t-\tau) + Z_C \cdot i_{km}(t-\tau) = v_m(t) + Z_C \cdot (-i_{mk}(t)) \tag{4.31}$$

Rearranging equation 4.31 gives the simple two-port equation for i_{mk}, i.e.

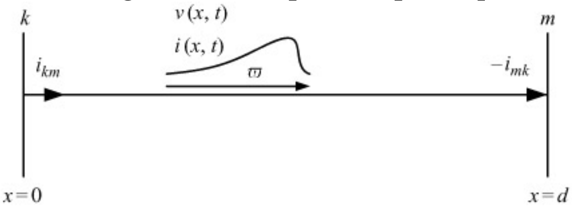


Figure 4.8: Propagation of a wave on a transmission line

$$i_{mk}(t) = \frac{1}{Z_C} v_m(t) + I_m(t-\tau) \tag{4.32}$$

where the current source from past History terms is:

$$I_m(t-\tau) = -\frac{1}{Z_C} v_k(t-\tau) - i_{km}(t-\tau) \tag{4.33}$$

Similarly for the other end

$$i_{km}(t) = \frac{1}{Z_C} v_k(t) + I_k(t-\tau) \tag{4.34}$$

where

$$I_k(t-\tau) = -\frac{1}{Z_C} v_m(t-\tau) - i_{mk}(t-\tau)$$

The expressions $(x - \omega t)$ = constant and $(x + \omega t)$ = constant are called the characteristic equations of the differential equations.

Figure 4.9 depicts the resulting two-port model. There is no direct connection between the two terminals and the conditions at one end are seen indirectly and with time delays (travelling time) at the other through the current sources. The past History terms are stored in a ring buffer and hence the maximum travelling time that can be represented is the time step multiplied by the number of locations in the buffer. Since the time delay is not usually a multiple of the time-step, the past History terms on either side of the actual travelling time are extracted and interpolated to give the correct travelling time.

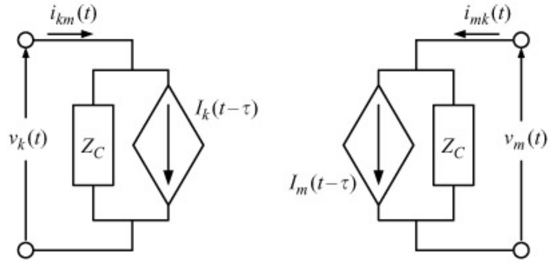


Figure 4.9: Equivalent two-port network for a lossless line

4.4. Network Solution

With all the network components represented by Norton equivalents a nodal formulation is used to perform the system solution.

The nodal equation is:

$$[G]\mathbf{v}(t) = \mathbf{i}(t) + \mathbf{I}_{\text{History}} \tag{4.35}$$

where:

- $[G]$ is the conductance matrix
- $\mathbf{v}(t)$ is the vector of nodal voltages
- $\mathbf{i}(t)$ is the vector of external current sources
- $\mathbf{I}_{\text{History}}$ is the vector current sources representing past history terms.

The nodal formulation is illustrated with reference to the circuit in Figure 4.10 [5] where the use of Kirchhoff's current law at node 1 yields:

$$i_{12}(t) + i_{13}(t) + i_{14}(t) + i_{15}(t) = i_1(t) \tag{4.36}$$

Expressing each branch current in terms of node voltages gives:

$$i_{12}(t) = \frac{1}{R}(v_1(t) - v_2(t)) \tag{4.37}$$

$$i_{13}(t) = \frac{\Delta t}{2L}(v_1(t) - v_3(t)) + I_{13}(t - \Delta t) \tag{4.38}$$

$$i_{14}(t) = \frac{2C}{\Delta t}(v_1(t) - v_4(t)) + I_{14}(t - \Delta t) \tag{4.39}$$

$$i_{15}(t) = \frac{1}{Z}v_1(t) + I_{15}(t - \tau) \tag{4.40}$$

Substituting these gives the following equation for node 1:

$$\left(\frac{1}{R} + \frac{\Delta t}{2L} + \frac{2C}{\Delta t} + \frac{1}{Z}\right)v_1(t) - \frac{1}{R}v_2(t) - \frac{\Delta t}{2L}v_3(t) - \frac{2C}{\Delta t}v_4(t)$$
$$= I_1(t - \Delta t) - I_{13}(t - \Delta t) - I_{14}(t - \Delta t) - I_{15}(t - \Delta t) \tag{4.41}$$

Note that [*G*] is real and symmetric when incorporating network components. If control equations are incorporated into the same [*G*] matrix, the symmetry is lost; these are, however, solved separately in many programs. As the elements of [*G*] are dependent on the time step, by keeping the time step constant [*G*] is constant and triangular factorisation can be performed before entering the time step loop. Moreover, each node in a power system is connected to only a few other nodes and therefore the conductance matrix is sparse. This property is exploited by only storing non-zero elements and using optimal ordering elimination schemes.

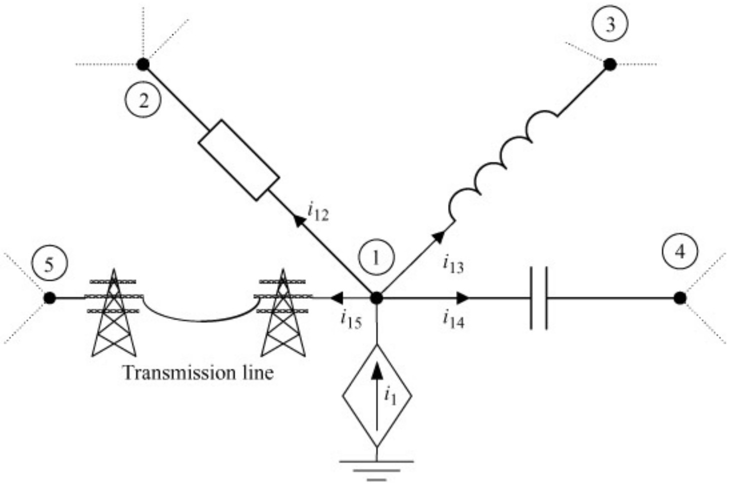


Figure 4.10: Node 1 of an interconnected circuit

Some of the node voltages will be known due to the presence of voltage sources in the system, but the majority are unknown. In the presence of series impedance with each voltage source the combination can be converted to a Norton equivalent and the algorithm remains unchanged.

Example: Conversion of voltage sources to current sources

To illustrate the incorporation of known voltages the simple network displayed in Figure 4.11 (a) will be considered. The task is to write the matrix equation that must be solved at each time point.

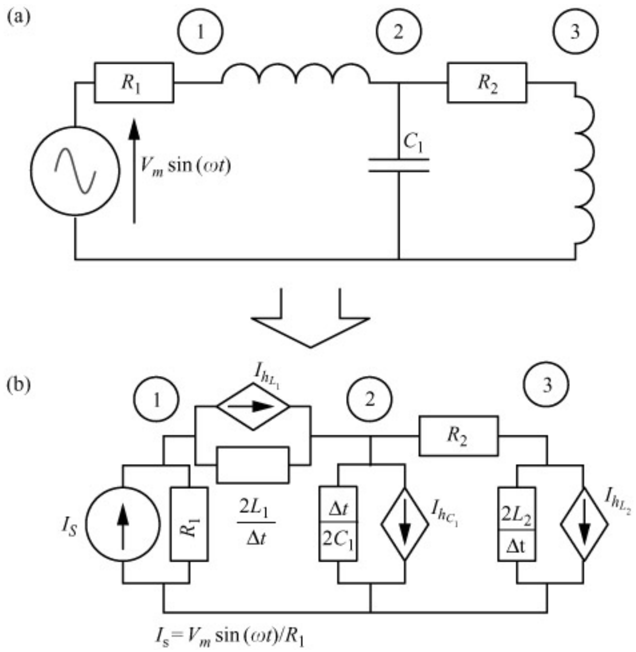


Figure 4.11: Example using conversion of voltage source to current source

Converting the components of Figure 4.11 (a) to Norton equivalents (companion circuits) produces the circuit of Figure 4.11 (b) and the corresponding nodal equation:

$$\begin{bmatrix} \frac{1}{R_1}+\frac{\Delta t}{2L_1} & -\frac{\Delta t}{2L_1} & 0 \\ -\frac{\Delta t}{2L_1} & \frac{\Delta t}{2L_1}+\frac{1}{R_2}+\frac{2C_1}{\Delta t} & -\frac{1}{R_2} \\ 0 & -\frac{1}{R_2} & \frac{1}{R_2}+\frac{\Delta t}{2L_2} \end{bmatrix} \begin{pmatrix} v_1 \\ v_2 \\ v_3 \end{pmatrix} = \begin{pmatrix} \frac{V_m \sin(\omega t)}{R_1} - I_{h_{L_1}} \\ I_{h_{L_1}} - I_{h_{C_1}} \\ -I_{h_{L_2}} \end{pmatrix} \tag{4.42}$$

Equation 4.42 is first solved for the node voltages and from these all the branch currents are calculated. Time is then advanced and the current sources representing History terms (previous time step information) are recalculated. The value of the voltage source is recalculated at the new time point and so is the matrix equation. The process of solving the matrix equation, calculating all currents in the system, advancing time and updating History terms is continued until the time range of the study is completed.

As indicated earlier, the conversion of voltage sources to Norton equivalents requires some series impedance, i.e. an ideal voltage source cannot be represented using this simple conductance method. A more general approach is to partition the nodal equation as follows:

$$\begin{bmatrix} [G_{UU}] & [G_{UK}] \\ [G_{KU}] & [G_{KK}] \end{bmatrix} \cdot \begin{pmatrix} \mathbf{v}_U(t) \\ \mathbf{v}_K(t) \end{pmatrix} = \begin{pmatrix} \mathbf{i}_U(t) \\ \mathbf{i}_K(t) \end{pmatrix} + \begin{pmatrix} \mathbf{I}_{U\text{History}} \\ \mathbf{I}_{K\text{History}} \end{pmatrix} = \begin{pmatrix} \mathbf{I}_U \\ \mathbf{I}_K \end{pmatrix} \tag{4.43}$$

where the subscripts U and K represent connections to nodes with unknown and known voltages, respectively. Using Kron's reduction the unknown voltage vector is obtained from:

$$[G_{UU}]\mathbf{v}_U(t) = \mathbf{i}_U(t) + \mathbf{I}_{U\text{History}} - [G_{UK}]\mathbf{v}_K(t) = \mathbf{I}'_U \tag{4.44}$$

The current in voltage sources can be calculated using:

$$[G_{KU}]\mathbf{v}_U(t) + [G_{KK}]\mathbf{v}_K(t) - \mathbf{I}_{K\text{History}} = \mathbf{i}_K(t) \tag{4.45}$$

The process for solving equation 4.44 is depicted in Figure 4.12. Only the right-hand side of this equation needs to be recalculated at each time step. Triangular factorisation is performed on the augmented matrix $[G_{UU}\ G_{UK}]$ before entering the time step loop. The same process is then extended to $\mathbf{i}_U(t)$ - $\mathbf{I}_{\text{History}}$ at each time step (forward solution), followed by back substitution to get $\mathbf{V}_U(t)$. Once $\mathbf{V}_U(t)$ has been found, the History terms for the next time step are calculated.

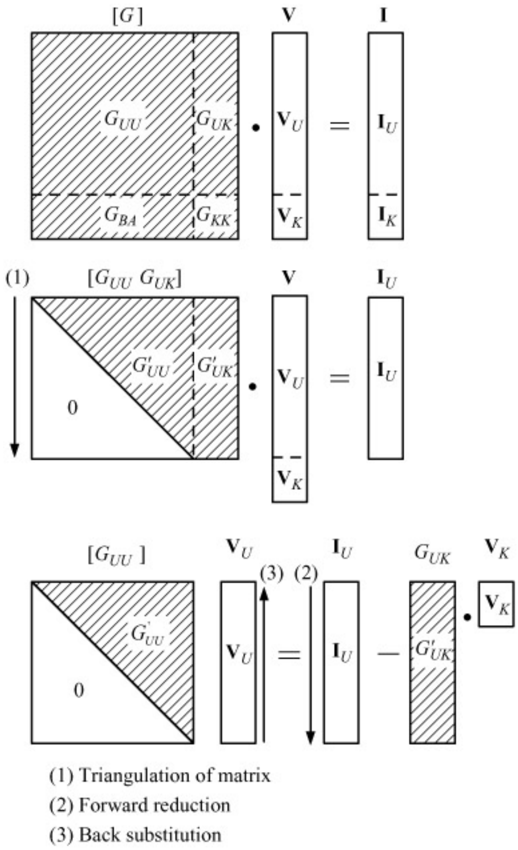


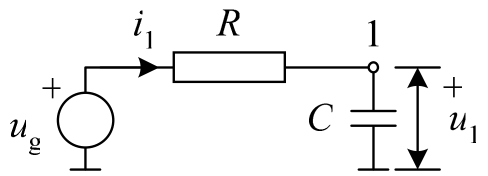
Figure 4.12: Network solution with voltage sources

4.5. Примери

Пример 4.1. RC колото од сликата 4.13 со параметри R = 1000 Ω и C = 100 μF е поврзано на идеален напонски генератор со напон u_g. Користејќи го методот на трапезна интеграција со временски чекор од 50 μs да се одреди напонот на јазелот 1 до временскиот момент t = 1 s за следните два случаи:

а) $u_g = 12$ V,

б) $u_g = 12 \cdot \sin(100\pi \cdot t)$ V.



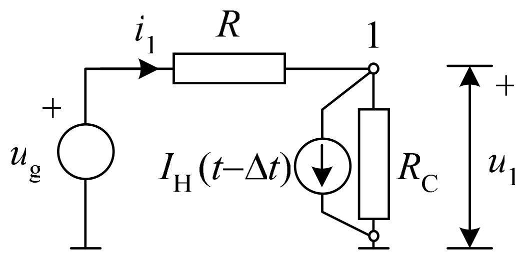
Слика 4.13. RC – коло

Решение

Еквивалентното коло за примена на правилото за трапезна интеграција е дадено на сликата 4.14. Параметрите на отпорникот R_C и струјниот генератор I_H со кои што е заменет кондензаторот ги пресметуваме според релациите (4.12) и (4.13) на следниот начин

$$R_C = \frac{\Delta t}{2C} = \frac{50}{2 \cdot 100} = 0,25\ \Omega\,, \tag{4.46}$$

$$I_H(t-\Delta t) = -i_1(t-\Delta t) - \frac{u_1(t-\Delta t)}{R_C}\,. \tag{4.47}$$



Слика 4.14. Еквивалентно коло за RC – колото од сликата 4.13

Според методот на независни напони за колото од сликата 4.14 можеме да ја напишеме следната равенка

$$G_{11} \cdot u_1 = i_{g1}\,, \tag{4.48}$$

односно

$$\left(\frac{1}{R} + \frac{1}{R_C}\right) \cdot u_1 = \frac{u_g}{R} - I_H(t-\Delta t)\,, \tag{4.49}$$

од каде што добиваме

$$u_1 = \frac{\frac{u_g}{R} - I_H\left(t - \Delta t\right)}{\frac{1}{R} + \frac{1}{R_C}}. \tag{4.50}$$

Струјата i_1 која што ни е потребна за пресметка на $I_H\left(t-\Delta t\right)$ изнесува

$$i_1 = \frac{u_g - u_1}{R}. \tag{4.51}$$

За да го пресметаме бараниот напон ќе ја користиме равенката (4.50) за сите временски моменти од 0 до 1 s со чекор од 50 μs при што во секој чекор вредноста на струјата $I_H\left(t-\Delta t\right)$ која што се однесува на еден чекор пред разгледуваниот ќе ја пресметуваме со помош на релацијата (4.47) одкако претходно ќе ја пресметаме струјата i_1 со помош на релацијата (4.51). Целата постапка е дадена во програмата Primer_4_1a.m која што го има следниот изглед

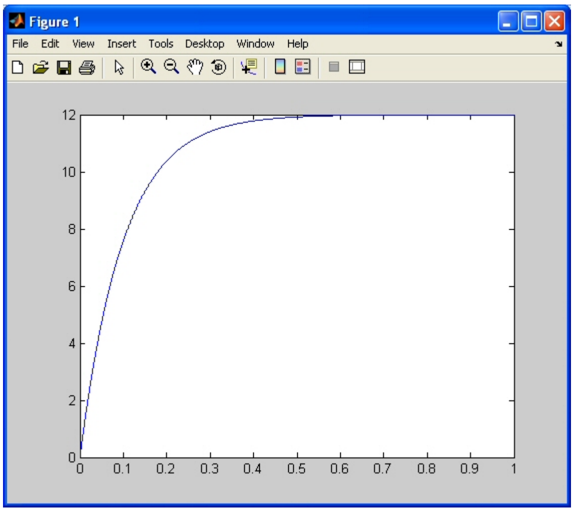
```
clear; clc;
dt = 50e-6;
Tkraj = 1;
R = 1000;
C = 100e-6;
Ug = 12;
RC = dt / (2*C);
N = Tkraj / dt;
t = zeros(N, 1);
v1 = zeros(N, 1);
i1 = zeros(N, 1);
IH = 0;
for i = 1 : N
    t(i) = i * dt;
    if i > 1
        IH = -i1(i-1) - v1(i-1)/RC;
    end
    v1(i) = (Ug/R - IH)/(1/R + 1/RC);
    i1(i) = (Ug - v1(i))/R;
end
plot(t,v1);
```

Во неа пресметките се изведуваат во рамките на еден for циклус со N чекори каде што е N = Tkraj / dt. Во првиот чекор вредноста на струјата IH не се пресметува, односно се зема дека таа е нула, а за останатите таа се пресметува со изразот (4.47). Со активирање на програмата го добиваме решението прикажано на сликата 4.15.

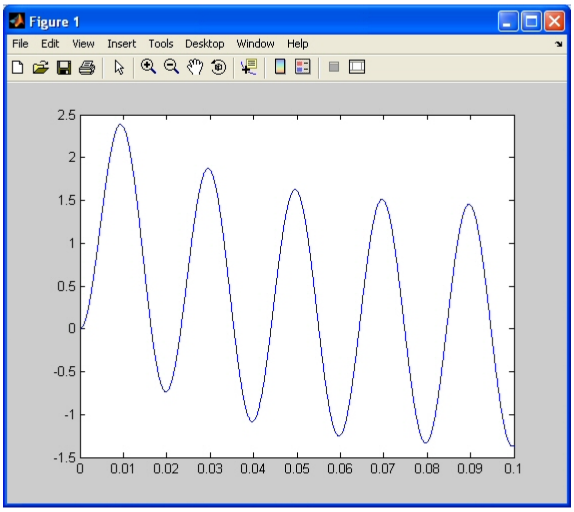
За случајот под б) бараниот напон е прикажан на сликата 4.16 а тој е пресметан со програмата која што е дадена во датотеката Primer_4_1b.m која што го има следниот изглед

```
clear; clc;
dt = 50e-6;
Tkraj = 0.1;
R = 1000;
C = 27e-6;
Ug = 12;
RC = dt / (2* C);
N = Tkraj / dt;
t = zeros(N, 1);
```

```
v1 = zeros(N, 1);
i1 = zeros(N, 1);
IH = 0;
for i = 1 : N
    t(i) = i * dt;
    if i > 1
        IH = -i1(i-1) - v1(i-1)/RC;
    end
    v1(i) = (Ug*sin(100*pi*t(i))/R - IH)/(1/R + 1/RC);
    i1(i) = (Ug*sin(100*pi*t(i)) - v1(i))/R;
end
plot(t,v1);
```

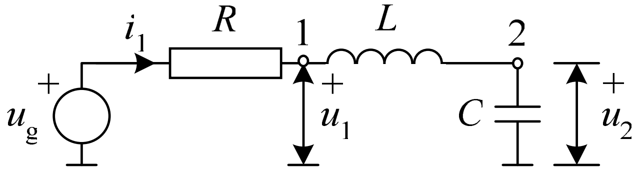


Слика 4.15. Напон на јазелот 1 во RC – колото за случајот под а)



Слика 4.16. Напон на јазелот 1 во RC – колото за случајот под б)

Пример 4.2. RLC колото од сликата 4.17 со параметри R = 3,6 Ω, L = 0,57 H и C = 100 μF е поврзано на идеален напонски генератор со константен напон u_g = 12 V. Користејќи го методот на трапезна интеграција со временски чекор од 50 μs да се одреди струјата во колото до временскиот момент t = 1 s.



Слика 4.17. RLC – коло

Решение

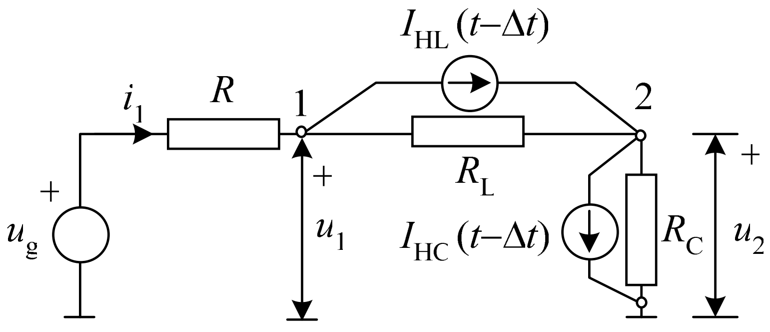
Еквивалентното коло за примена на правилото за трапезна интеграција е дадено на сликата 4.18. Параметрите на отпорникот R_C и струјниот генератор I_{HC} со кои што е заменет кондензаторот ги пресметуваме според релациите (4.12) и (4.13), додека параметрите на отпорникот R_L и струјниот генератор I_{HL} со кои што е заменет калемот ги пресметуваме според релациите (4.6a) и (4.6b) на следниот начин

$$R_C = \frac{\Delta t}{2C} = \frac{50}{2 \cdot 100} = 0,25\ \Omega, \tag{4.52}$$

$$I_{HC}\left(t-\Delta t\right) = -i_1\left(t-\Delta t\right) - \frac{u_2\left(t-\Delta t\right)}{R_C}, \tag{4.53}$$

$$R_L = \frac{2L}{\Delta t} = \frac{2 \cdot 0,57}{50 \cdot 10^{-6}} = 22800\ \Omega, \tag{4.54}$$

$$I_{HL}\left(t-\Delta t\right) = i_1\left(t-\Delta t\right) + \frac{u_1\left(t-\Delta t\right) - u_2\left(t-\Delta t\right)}{R_L}. \tag{4.55}$$



Слика 4.18. Еквивалентно коло за RLC – колото од сликата 4.17

Според методот на независни напони за колото од сликата 4.17 можеме да ја напишеме следната равенка

$$\begin{bmatrix} G_{11} & G_{12} \\ G_{21} & G_{22} \end{bmatrix} \cdot \begin{bmatrix} u_1 \\ u_2 \end{bmatrix} = \begin{bmatrix} i_{g1} \\ i_{g2} \end{bmatrix}, \tag{4.56}$$

каде што

$$\boldsymbol{G} = \begin{bmatrix} \frac{1}{R} + \frac{1}{R_L} & -\frac{1}{R_L} \\ -\frac{1}{R_L} & \frac{1}{R_L} + \frac{1}{R_C} \end{bmatrix}, \tag{4.57}$$

$$\boldsymbol{i}_g = \begin{bmatrix} \frac{u_g}{R} - I_{HL} \\ I_{HL} - I_{HC} \end{bmatrix}. \tag{4.58}$$

Напоните на независните јазли ги добиваме со решавање на системот равенки (4.54)

$$\begin{bmatrix} u_1 \\ u_2 \end{bmatrix} = \boldsymbol{Z} \cdot \boldsymbol{i}_g, \tag{4.59}$$

каде што

$$\boldsymbol{Z} = \boldsymbol{G}^{-1}. \tag{4.60}$$

Струјата i_1 која што ни е потребна за пресметка на $I_{HL}(t-\Delta t)$ и $I_{HC}(t-\Delta t)$ изнесува

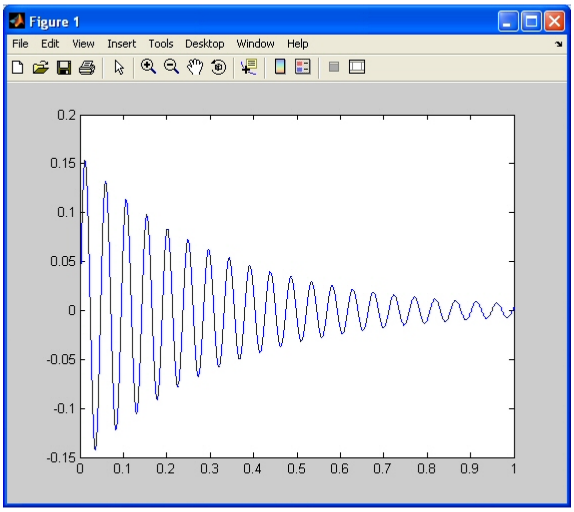
$$i_1 = \frac{u_g - u_1}{R}. \tag{4.61}$$

За да ги пресметаме напоните на независните јазли ќе ја користиме равенката (4.59) за сите временски моменти од 0 до 1 s со чекор од 50 μs при што во секој чекор вредностите на струите $I_{HL}(t-\Delta t)$ и $I_{HC}(t-\Delta t)$ кои што се однесуваат на еден чекор пред разгледуваниот ќе ги пресметуваме со помош на релациите (4.53) и (4.55) одкако претходно ќе ја пресметаме струјата i_1 со помош на релацијата (4.61). Целата постапка е дадена во програмата Primer_4_2.m која што го има следниот изглед

```
clear; clc;
dt = 50e-6;
Tkraj = 1;
R = 3.6;
L = 0.57;
C = 100e-6;
Ug = 12;
RL = 2*L / dt;
RC = dt / (2*C);
G = [
    1/R + 1/RL  -1/RL
    -1/RL       1/RL + 1/RC
    ];
Z = inv(G);
N = Tkraj / dt;
t = zeros(N, 1);
v1 = zeros(N, 1);
v2 = zeros(N, 1);
i1 = zeros(N, 1);
IHL = 0;
IHC = 0;
for i = 1 : N
```

```
    t(i) = i * dt;
    if i > 1
        IHL = i1(i-1) + (v1(i-1) - v2(i-1))/RL;
        IHC = -i1(i-1) - v2(i-1)/RC;
    end
    D = [
        Ug/R - IHL
        IHL - IHC
        ];
    Rez = Z * D;
    v1(i) = Rez(1);
    v2(i) = Rez(2);
    i1(i) = (Ug - v1(i))/R;
end
plot(t,i1);
```

Во неа пресметките се изведуваат во рамките на еден for циклус со N чекори каде што е N = Tkraj / dt. Во првиот чекор вредноста на струите IH и IC не се пресметуваат, односно се зема дека тие се еднакви на нула. Со активирање на програмата го добиваме обликот на струјата прикажан на сликата 4.19. Доколку во командниот простор напишеме plot(t,v1) или plot(t,v2) ќе го добиеме и обликот на напоните на јазлите 1 и 2.



Слика 4.19. Струја во RLC – колото

□ □ □